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"Fractal as an approach to comprehend decorative pattern designs: A case of Humayun Tomb, India."

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ABSTRACT

Mathematical theories of fractal geometry have been extensively studied since 1975 to analyse complex works such as city planning, art, and architecture, focusing on self-similarity at various scales. Mughal art and architecture are recognised for the stunning formations created by patterns, geometry, and scaling, all of which maintain a consistent style. Yet, they do not explore the interpretation of pattern designs from a fractal perspective. Given this perspective, the paper explores how geometry serves as a central organising principle that ties together various aesthetic elements in Mughal art and architecture using a fractal methodology. This paper aims to examine the connection between fractal geometry and continuous decorations in Mughal art and architecture, using "Humayun's Tomb" as a case study. The paper discusses how fractal aesthetics in Mughal patterns embrace a distinct concept of self-similarity, integrating motifs at various scales within patterns. The religious beliefs about Paradise are intricately woven into the structure through its decorative elements and stylized expression. Utilising a literature review of historical texts and research papers, this paper delves into religious ontology and a fractal approach. It employs the box-counting method to perform a fractal-visual analysis of the

architectural structure and decorative components of the Humayun's Tomb and its Landscape. This study will suggest the geometric physical implications as a method to decode structural designs and their metaphysical meanings, offering a fresh outlook on Mughal art by utilising "Fractal Geometry" as an interpretive tool.

Keywords: *Mughal-Architecture &Fractal, Humayun's Tomb, Box-Counting Method.*

Geometry plays a substantial role in Mughal Art's spatial and ornamental systems. Geometry is used as an ordering paradigm and unifier of all aesthetic schemes in Mughal architecture. The structural order of intricately balanced symmetry ordered repetition, and rhythmic patterns, are the remarkable attributes of Mughal architecture. The spectacular formations choreographed by patterns and scaling follow a singular unity of style, manifesting progressive refinement of the stylistic ideals. Most Mughal architectural expressions (religious/non-religious) are ordered from religious symbolism and adorned with pattern designs. These together manifest exceptional decorative creations of outstanding exhibit value. Mughal art and architecture in India is a cultural culmination of the Islamic tradition with regional traditions of India. This assimilation and internalization of every regional and local tradition led to the formation of a unique cultural expression while keeping its religious underpinnings intact. The researcher's opinions that it became possible as the attributes of fractal geometry and Islamic theological concepts shared few similar characteristics. The patterns and building designs in global Islamic counterparts, the fractal components were retained even after adaptation to regional and local designs.

Fractal geometry elucidates intricate structures and patterns that were previously unexplainable. Fractal geometry has been used in city planning, abstract art, and architecture since 1975 to analyze complex works for self-similarity at different scales. Fractal is not used to explore Indian Mughal art and architectural ornamental pattern designs. Also, when religious symbolism is combined with style, this research seeks to understand the relationship between fractal geometry and continuous colorful, symbolic manifestations in Mughal art and architecture. The study explores Mughal patterns' fractal aesthetics, which incorporate motifs at many, often infinite scales to achieve self-similarity. The study examines Mughal art's ordered repetition, radiating structures, and generic patterns. The paper uses fractal as a conceptual framework and tool to examine and decipher geometrical manifestations, addressing the metaphysical connotations of structure designs. Through "Fractal Geometry." the method will experimentally analyse and understand Mughal art.

The examination and identification of Mughals art's fundamental geometrical forms and beautiful symbolic expressions can help identify and associate fractal structures across architectural material expressions. The ubiquitous'self-similarity' of Mughal architecture's aesthetic quality makes the researcher cautiously believe that fractal geometry serving the same attribute can be used to study Mughal art and architecture from a new perspective.

Fractal geometry is a theoretical framework that defines forms and patterns previously considered too complex to describe. Since 1975, mathematical theories of fractal geometry have been widely adopted and explored in fields of city planning, abstract art, and architecture to facilitate an analysis of complex works with an eye for self-similarity at different scales. The application of fractal is not attempted to study decorative pattern designs in general and

specific to Indian Mughal art and architecture. That too when the religious symbolism is married to stylistic expressions. In this backdrop, the present research attempts to study and shed more light on the nature of the relationship between fractal geometry and continuous decorative symbolic expressions of Mughal art and architecture. The study aims to dwell into the fractal aesthetics of Mughal patterns that pursue a unique notion of self-similarity, simultaneously incorporating motifs at multiple—often, infinite scales within patterns. The study is trying to dive into the distinguishing features of Mughal art's ordered repetition, radiating structures, and generic patterns. The paper explores and deciphers geometrical manifestations simultaneously using fractal as a conceptual framework and a method, considering the possible alignment between structural designs and their metaphysical connotations. The method will provide an experimental perspective to approach, analyze and understand Mughal art through the lens of "Fractal Geometry."

The analysis and identification of Mughals art's primary geometrical forms and decorative symbolic expressions will enable identifying and associating a set of fractal structures across material expressions in architecture. Considering the ubiquitous 'self-similarity', an essential part of the Mughal architecture's aesthetic quality, the researcher cautiously considers that the fractal geometry serving the same attribute can be an effective method to study Mughal art and architecture providing a new architecture perspective.

A GLANCE INTO RELATED LITERATURE:

Geometry as a concept has been widely used and always attributed as an essential element in different art forms. However, the challenge was to understand the chaotic geometrical shapes

overlaid with multiple geometrical elements. The fractal is a theoretical framework that defines united forms and patterns previously considered too complex to describe. 'The fractal geometry of nature, a book by 'Benoit B. Mandelbrot,' brought the revolution opening a new era of geometry-based study in diverse areas of sciences, philosophy, and art. It promotes a mathematical and philosophical synthesis. (Mandelbrot). From 1975 onwards, fractal geometry has deeply influenced landscape perception, architecture, and technology (Agnès Patuano & M. Francisca Lima) along stimulating development in several fields such as architecture, urban planning, biology, medicines, art, and literature through enabling understanding of chaos. Mandelbrot cautions us, "Fractals will make you see everything differently....You risk the loss of your childhood vision of clouds, forests, galaxies, leaves, flowers, rocks, mountains, torrents of water, carpets, bricks, and much else besides (Mandelbrot)". Antonio Di Ieva, considered it a universal pattern language as the form it describes can be found in every living thing (Ieva). Fractal Geometry, a mathematical innovation, explores the concept of infinite scale by utilizing digital simulations. It has progressively integrated pictures from other fields, such as Art and Science. Specific theories in fractal properties, such as Iterated Function Systems (IFS), talk about the aesthetic properties regarding proportion, rhythm, and symmetry. The representation of fractality is not only limited to one specific art form, but one can also see the attempt to create order through the chaos of the forms throughout the human past in human creations, from ancient Hindu temples to the interior of the central dome of the Selimiye Mosque in Turkey, to Australian aboriginal artworks dating back tens of thousands of years. The depiction of fractal forms can be analyzed in various historical architecture.

The geometry used fits well in the theological ontology of Islam. The Persian gardens and Charbagh share as a concept symbolizing the importance of sharing water and the concept of heaven (Agnès Patuano & M. Francisca Lima) Therefore, fractal geometry has been a key utility for ornamentation and composition throughout the Islamic expressions. The concept of infinity, which signifies the existence of fractals, is prominently featured in nearly all forms of Islamic art as a crucial element, including beautiful symbolic representations. Fractal Geometry is founded on the concept of the "principle of repetition" of geometric shapes, which allows an item to retain and reproduce its shape when magnified at any level. The proposal presents the concept of "Infinity" in a systematic yet non-continuous manner, referring to the same pre-existing "forms". Hence, it becomes evident that nature demonstrates its artistic expression through a consistent yet unpredictable pattern. The scale invariance qualities of self-similarity imply that the object's irregularity remains the same at different sizes of creation and perception. This revised Geometry provides a clearer representation of the correlation between the object and the observer's "scaled position". Within a fractal structure, the presence of "hidden symmetry" is not limited to just one aspect. Hidden within the virtue of self-similarity lies a new idea of "symmetry and harmony" that expands upon the classical Greek doctrine of "harmony and proportions." Architecture exhibits a multitude of recurring patterns. "The presence of fractal geometry in fine arts and city planning can be demonstrated by identifying fractal characteristics in paintings and cities"(Agnès Patuano & M. Francisca Lima). In the books "Fractals and Fractal Architecture" (Lorenz) and "Fractal Geometry in Architecture and Design" (Bovill), the Box-counting dimension method has been suggested to analyze the level of fractality in architectural forms.

The paper focuses on a theoretical approach to integrate multiple geometry components to provide a theoretical framework. It will contribute to analyzing the aesthetical and compositional characteristics of the continuous symbolic representation of patterns in Mughal art and architecture. A compositional fractal analysis encompasses two methods: visual fractal analysis and dimension fractal analysis. This study examines the coherence between many elements such as landscape plan, section, elevation, floor plan, and ornamental motif (M.Y. Shishin; Khalid J.Aldeen Ismail). A spatial fractal has been introduced to understand the high homogeneity and harmony in the façades of Islamic buildings, from the Mughal period. The approximation values of the fractal properties have been observed in this particular architecture (Asia Jabeen). Perhaps one of the well-recognized examples of Mughal architecture is the Taj Mahal. Throughout the monument, one can see 6-point, 8-point, 10-point, 12-point, 16-point, and other complex patterns. The aerial view represents the tomb's octagonal cross-section structure that symbolizes the physical world and the path to heaven.

Mughal architects used fractal systems in numerous applications, reflecting the great importance of fractal methods and techniques within the architectural design process. It starts with tiny details, like mosaic ornaments and Arabesque carving, continued in entry gateways, doorways, arches, etc., to city planning in fractals, which are visible in the urban morphology, fabric, and context. The planning of the urban Mughal city is based on a hierarchal roads network and a set of residential clusters divided into a dense fabric of small residential buildings (Elgohary). It demonstrates the unity and diversity of the Islamic city, which appeared equivalent at any magnification.

Art is often regarded as a means of discovering the fundamental principles of beauty and harmony inherent in the natural world. Chaos and fractal geometry can be utilised to elucidate and substantiate the principles of beauty (Lorenz). The geometrical tools will further analyze Mughal art's decorative symbolic expression to provide a language for interpretation of aesthetics. Therefore, the symbolic expression of Mughal art has been formalized in endless repetition. This manifestation of religious ontology will be observed to analyze the relationship between the fractal principles and theology of stylized symbolism in Mughal art.

Art of geometry "Harmonious Infinity of designs in Mughal Architecture." Tomb architecture: a manifestation of the philosophy of Paradise (ontological perspective) Emanation is the key concept to reveal the fundamentals of Islamic Ontology. Emanation, from the Latin emanare meaning "to flow from" is the idea that all things are derived from the First Being, or Principle (Güney). The tomb architecture of Mughal's had been derived from the idea of "paradise." The concept of Paradise was originally established by the Persians, who associated it with a physical garden that symbolised the metaphysical idea of the afterlife. The term "pairidaēza" in Persian refers to a "walled garden" and was conceived as a physical enclosure that symbolises a heavenly location (Roselyn D'Mello, "Return of the Tomb" Art Illustrated/24sep2020) The idea of gardens, Charbagh's architecture, and glorified decoration all together become a successful effort to create a heavenly atmosphere. Being a monotheistic religion, Islam discarded all other deities and avoided depicting realistic humans and Animal figures. Artisans in the Islamic world drew inspiration from the classical heritage of ancient Greek-Roman, Sasanian, and Iranian art. They

then developed a unique style of decorating using just a compass and ruler. Patterns are predominantly present in the majority of Mughal architecture and serve as a means to exalt God. These patterns exhibit symmetry and repetition, together with intricacy, allowing for limitless extension and creating visually appealing designs. The intricate patterns and vivid hues also attract the observer towards a state of serene boundlessness, evoking reflection and oneness.

The Mughals were the descendants of Timurids, and they have been heavily promoted with the aid of the Persian-fashion gardens found in imperative Asia. The style of architecture explicitly used in Mughal's architecture was borrowed from the Persian garden's architecture; however, modified in line with the characteristics and site topography of the newly conceived gardens. It is logical that the Persian architect Mīrak Mīrzā Ghiyās was hired in 1565 by Hamida Banu Begum, the widow of Humayun, the second Mughal emperor of India, to construct his mausoleum near the dargah and shrine of the Sufi saint, Chishti Shaikh Nizamuddin Auliya. He reproduced the concept of the "paradaida", providing the Indian subcontinent with its inaugural instance of a sophisticated garden mausoleum (D'Mello).

The significant role of religious symbolism in Mughal architecture is proven beyond doubt by many studies. Islam being an aniconistic⁷ religion, non-figurative ornamentation has always been an inborn behavior throughout the Islam constructions. Mughal patrons and their artists and artisans abide the same, and geometric patterns are widely seen in their art and ornamentation (Güney). The universe's infinite nature is the common and essential characteristic of Islam's theology and the fractal. The

⁷ *aniconism, in religion, opposition to the use of icons or visual images to depict living creatures or religious figures*

symbolic depiction of the 'infinite' in this Mughal architecture repeats itself forever and ever through beautifully ordered symmetry in all structural components. Every structural detail depicted something or other from the ontological aspect of the religion as a symbolic expression. It seems to serve the needs of aesthetics as well the religious symbolism impartially. In this sense, the aesthetics, fractal geometry, and religion together serve as corners of a triangle called Mughal Art and architecture. Considering this, the research adopted to build a fractal theoretical perspective for Mughal period art expressions through finding answers for the following research questions:

RQ1: Nature of relationship between fractal geometry and repetitive decorative symbolic expressions in Mughal architecture?

RQ2: How is the religious ontology integrated and epitomized into the structure through its decorative components and stylized expression?

In view of the research question, Firstly, some necessary notions of fractal geometry will be discussed, followed by a description regarding the creation of chahár-bágh gardens architecture with a particular example of the Humayun tomb plan. The application of fractal geometry in the Humayun tomb can help understand the philosophy of design fundamentals in symbolic decorative expressions of Mughal architecture. Through the elevation, geometric patterns in ornamentations will validate that fractal geometry also exists along with linear and Euclidian geometry.

METHODOLOGY:

The present study uses both qualitative and quantitative methods to examine both research questions. The literature review of historical texts and research papers is used to understand

religious ontology. Secondly, the box-counting method is used to conduct the fractal-visual analysis of the architectural structure and the decorative components of the Humayun's Tomb and its Landscape. The collected qualitative data will be documented and analyzed to understand fractal geometry proportions in the case study, thereby in the Mughal architecture.

BOX COUNTING METHOD:

The box-counting method is probably the most well-known approach in any discipline for determining an image's fractal dimension, which quantifies characteristic visual and structural complexity. Following the earlier work by (Asia Jabeen) and (M.Y. Shishin; Khalid J.Aldeen Ismail) this paper uses a image analysis software program - "Image J," which allows one to make a fractal analysis of projections of building.

The derivative value of the fractal dimension indicates the details in the form of the particular part of the architecture. The higher the fractal dimension value, the more details in the form. It shows that the fractal dimension of any form describes the progression of the details (Asia Jabeen). The Box Counting Method in Image j software follows this process

- **Select the form to measure the fractal dimension**
- **Convert the image into a binary image (Select the process- Binary)**
- **Analyses- Tools- Fractal Box Counting**
- **Select the box sizes**
- **Tabulate the results as per the table.**
- **The fractal dimension "D" will be determined**

What is Fractal?

Fractal is a component in any of the highly irregular shapes or curves for which any particular selected part is similar in shape to an existing larger or smaller part when its magnification is equalized at the same size. The fractal pattern is a kind of equation occurring all around us in nature, and these patterns find constancy in randomness, Therefore, a tree can be divided into parts, and each successive part is a very similar to the whole structure, sometimes identical or a similar copy of the whole structure. A fractal is a potentially never-ending pattern, created by repeating a simple process over and over again.

Fractal Dimension: Fractal shapes have many essential characteristics; one of them is the fractal dimension. This character explains that fractal dimension is not an integer but ranges between 1 and 2, whereas Euclidean geometry deals with the integers. Mandelbrot found that fractal shapes can be described through an actual number that has a value between 1 and 2, which explains the meander and complexity. It is termed as "Fractal Dimension" (Asia Jabeen) To check the level of fractality in architectural forms, "A box-counting" method has been proposed by W. Lorenze (2002) and C. Bovil (1996) in their research. Their work had proposed the idea of the box-counting method, which has often been used to calculate computer programs. In their work, W. Lorenze and C. Bovil have suggested the Box-counting dimension method to check the level of fractality in architectural forms.

Fractal in Mughal architecture:

Fractal geometry in architecture is not a recent phenomenon, but unconsciously, fractal components have been widely used in various traditional architecture for centuries. Over time, many

have analyzed that in architecture. The history of self-similarity in architecture can be dated back to the early Hindu temples and even before. However, the concept of fractal and self-similarity in the earlier examples of architecture are unconscious efforts. The influence of religious aniconism is a prominent element of most Mughal architecture and art.; however, in ornamentation, artists and artisans preferred geometric patterns due to the strong prohibition of idolatry in the religious beliefs. Though the Architectural forms are artificial and thus very much based in Euclidean geometry, but we can find some fractal components in the architecture, too (SALA). Within its foundation of abstract mathematics, fractal can be applied to aid in design. This comes in many layers; some of the most fundamental architectural forms (floor, wall, roof). If we compare the apparent aspect of architecture, fractal can be directly related to self-similarity, organization, and complexity. According to Nicoletta Sala, "We can divide the fractal analysis in architecture into the two categories, little scale analysis and large scale analysis. This can be utilized in the methods of aesthetic appeal or even to the point where it essentially becomes a technical integration such as structure.

Case of Humayun's Tomb, Delhi, India:

Humāyūn's Tomb, is one of the earliest extant examples of the garden tomb characteristic of Mughal architecture, situated in Delhi, India. This tomb architecture is the template for what Mughal architecture became later. It stands amidst a large square garden, covered with walls with the gateway to the south and west. The structure includes four ivans, the four-grand opening on four sides, four corners of the rooms, and a dome on top. This kind of plan is often discussed as a "Hasht Bihistt Plan" هشت بهشت -Eight Paradises. This plan is literally translated as eight Paradise, and there are several buildings in Isfahan and Shiraz that use a similar

planning principle. Where four rooms in four corners, four portals in the centers and then the space enclosed in the middle. The eight divisions and frequent octagonal forms of this plan represent the eight levels.

Humayun tomb is set on an enormous platform of great height. In the middle similarly like Babar's garden, there is also an arrangement of channels that divide the garden in to many quarters. The aerial view of Humayun's Tomb testifies to the conscious and extensive use of geometry. The large basement story on which the monument is kept, almost like a pedestal, is used to keep an object on it. It is one of the essential features of Mughal architecture.

The funneled-shaped gate was intentionally built to emancipate the sense of grandeur and infiniteness as narrow and crunched gate open to a huge crafted space playing an optical illusion on the visitor's minds. By making viewers funnel through relatively narrow gates, it surprises the viewer when they emerge on the other side. They suddenly see the canvas of their vision bursts open of its temporary confines and see the garden and the red sandstone mausoleum before them. Architecture symbolizes the ancient idea of the confrontation between light and darkness, the earth, and the cosmos' order in all the world's religions and philosophies.

Use of Fractal geometry in Humayun tomb:

Measurement of Fractal Dimension in Humayun's Tomb architecture will be applied on five different levels: landscape, elevation plan, building structure, ornaments, lattices patterns in two-dimensional spaces. Humayun's Tomb is the finest example of Mughal Architecture and expresses fractal geometry, a similar

“Charbagh” plan was followed in all later constructions of Mughal architectures.

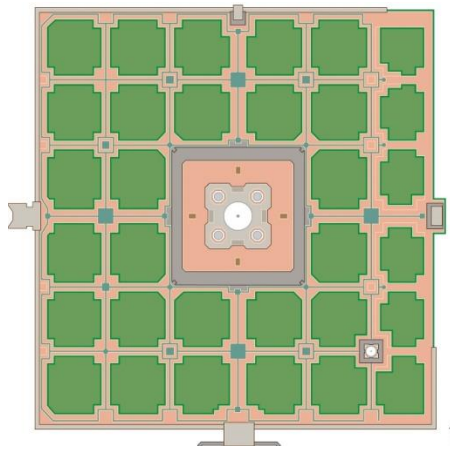


Table 1: Landscape: Level of fractal dimension in projections of the Humayun's Tomb

Grid Size	Number of Boxes
C 20	4363
C 40	1180
C 80	323
C 160	89
D	1.872

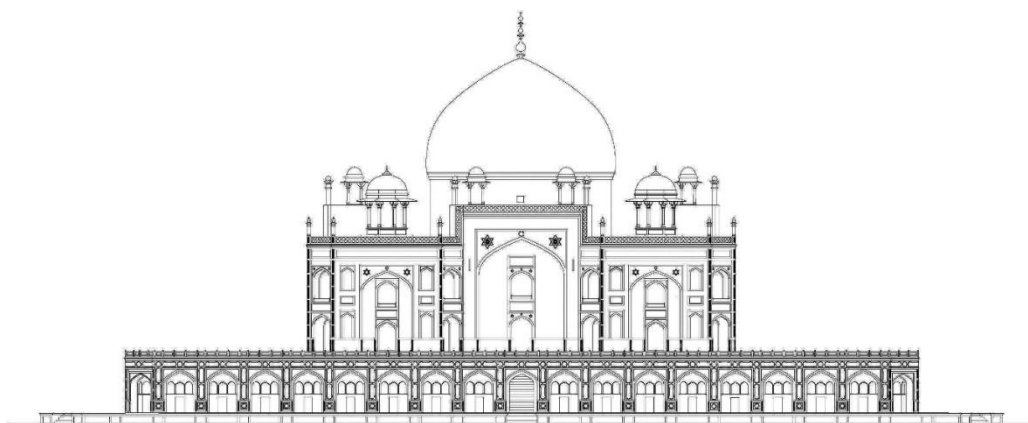


Table 2: Elevation: Level of fractal dimension in projections of the Humayun's Tomb

Grid Size	Number of boxes
C 20	2190
C 40	635
C 80	178
C 160	60
D	1.740

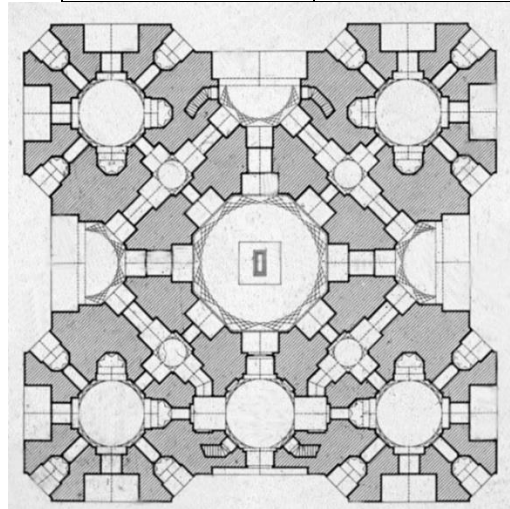


Table 3: Building:: Level of fractal dimension in projections of the Humayun's Tomb

Grid Size	Number of Boxes
C 20	1202
C 40	348
C 80	98
C 160	25
D	1.859

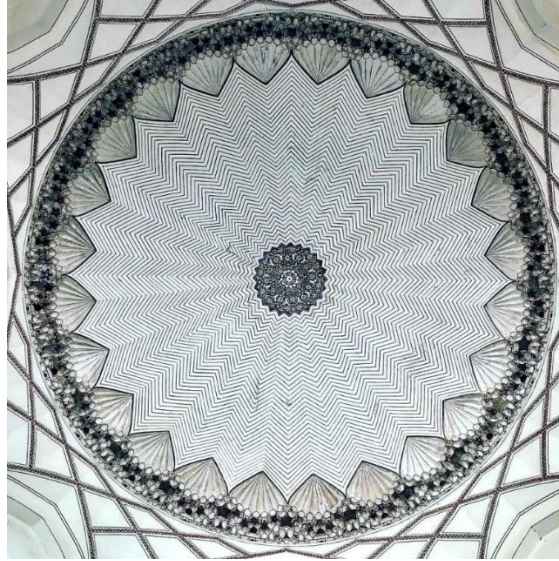


Table 4: Ornament: Level of fractal dimension in projections of the Humayun's Tomb

Grid Size	Number of Boxes
C 20	2206
C 40	571
C 80	144
C 160	36
D	1.980

For The jālis patterns:

The lattice screens, or 'jālis,' are an essential feature of this particular Mughal architecture. Filtering light through Jalis and the ambiance inside are the most elaborate design features, particularly in Mughal buildings. The interplay of light, shadow, and pattern has always been one of the primary structures based

on geometry. In articulating fractal geometry in Mughal architecture, these jālis patterns have an essential role.

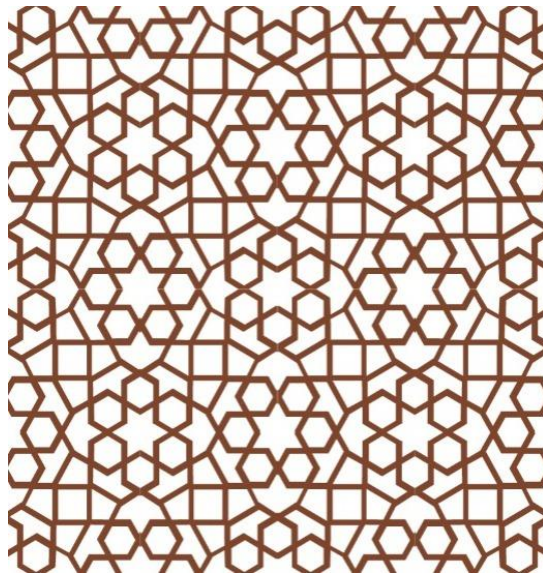


Image source:

https://commons.wikimedia.org/wiki/File:Portion_of_Pattern_of_Jali_from_Humayun%27s_Tomb.svg

Table 4: Ornament: Level of fractal dimension in projections of the Humayun's Tomb

Grid Size	Number of Boxes
C 20	739
C 40	196
C 80	49
C 160	16
D	1.859

The fractal dimension quantifies the degree of complexity and irregularity. The higher value of fractal dimension is a result of the

increased intricacy in the architectural structures. Buildings typically have a fractal dimension ranging from one to two. The design of Humayun's tomb clearly exhibits the presence of fractals in its horizontal spatial divisions, as evidenced by its characteristics.

The level of fractal intricacy affects the aesthetic values of the particular architectural component. The tenets of Islamic Ontology are based on the concept that God is the eternal and ultimate being. It believes that the existence of other beings is an illusion that came to reality by the emanation of God's supreme oneness. A fractal pattern is the one in many, and many in one as God and the creation is, according to Islamic Ontology. One staring at a fractal pattern can see how an archetype vitalizes the whole and how the whole reflects the archetype; this is the manifestation of Islamic ontology.

CONCLUSION :

This study briefly explores the connection between fractal theory and religious ontology in Mughal architecture. It aims to demonstrate the crucial importance of fractals in Mughal architecture. This study aimed to briefly discuss fractals and Mughal architecture. This inter-relationship has deepened our understanding of the vast structure, layer by layer. Each structural element of the tomb adheres to fractal forms. This paper is an attempt to emphasise them. Nevertheless, there is a significant opportunity to examine them more closely. In general, using fractals to understand decorative pattern designs, like those seen in the Humayun Tomb in India, provides a unique viewpoint that connects mathematics and aesthetics. Through examining the complex geometries and self-replicating structures found in these

patterns, researchers can uncover the inherent beauty and complexity that characterise these architectural masterpieces.

Mughal architecture offers a plethora of intricate details to captivate the observer. The structure's fractal characteristics highlight the intricate design of Humayun's Tomb as a well-organized and complex masterpiece. This specific Mughal architecture exemplifies a meticulous process of refinement to achieve its ultimate design. Hence, it serves as a remarkable illustration of architectural design rooted in fractal geometry, intertwining philosophical ideas of heaven, infinity, and related concepts.

Above, it is shown in the paper that the structure of Humayun's Tomb displays numerous variations due to its fractal features. The design highlights several fractal characteristics, including the emergence of an archetype form. The self-similar branched shapes, rhythm, and hierarchy of scaled spaces all point to fractal characteristics.

In future studies, researchers will explore fractal geometry on 3-D architectural forms, floor patterns, opening scales, ornament scales, and the subdivision of arches and windows.

References:

Agnès Patuano & M. Francisca Lima. "The fractal dimension of Islamic and Persian fourfolding gardens." Humanities and Social Sciences Communications (2021).

Asia Jabeen. "The Spatial Analysis of Fractal Geometry in Mughal Architecture: A case of Wazir Khan Mosque Lahore, Pakistan." International Conference on Sustainable Development in Civil Engineering, MUET, Pakistan, 2019.

Bovill, Carl. Fractal Geometry in Architecture and Design. Birkhäuser, Boston, MA, n.d.

Branka Spehar; Richard P. Taylor. "Fractals in Art and Nature: Why do we like them?" Proceedings of SPIE - The International Society for Optical Engineering. March 2013.

D'Mello, Rosalyn. "Return of the Tomb." 24 September 2020. logoAI | Arts Illustrated | An Indian Based Arts And Design Magazine. <<https://www.artsillustrated.in/architecture/return-of-the-tomb/>>.

Elgohary, Amr. "The Transformation of Using Fractal Forms Between Islamic and Digital Architecture from a Sustainable Approach." The Academic Research Community publication (ARChive) 3.3 (2019).

Gombrich, E. H. Art and Illusion : A Study in the Psychology of Pictorial Representation. Phaidon Press,, 1977.

Güney, Öncü. "Fractal Geometry and Islamic Ornamentation." (n.d.).

Ieva, Antonio Di. "The Fractal Geometry of the Brain: An Overview. ." Ieva, Antonio Di. *The Fractal Geometry of the Brain*. New York: Springer, New York, NY, 2016. 3-12.

Jabeen, Asia. "The Spatial Analysis of Fractal Geometry in Mughal Architecture: A case of Wazir Khan Mosque Lahore, Pakistan." *International Conference on Sustainable Development in Civil Engineering, MUET, . Pakistan , 2019.*

Kocic, Ljubisa M. "ART ELEMENTS IN FRACTAL CONSTRUCTIONS." *eLibrary of Mathematical Institute, of the Serbian Academy of Sciences and Arts 4.1 (2002).*

Lorenz, Wolfgang E. "Fractals and Fractal Architecture." 2003.

M.Y. Shishin; Khalid J.Aldeen Ismail. "A method of compositional fractal analysis and its application in Islamic architectural ensembles." n.d. researchgate.

Mandelbrot, Benoit B. the fractal geometry of nature. Newyork : W.H. freeman and company , 1982.

Morgan, Steev. Applying Chaos Theory to Artistic and Cultural Practice. 2007. <<https://steev.ca/chaos-in-art-and-cultural-practice/>>.

SALA, NICOLETTA. "FRACTAL MODELS IN ARCHITECTURE: A CASE OF STUDY." *International Conference on "Mathematics for Living". November 2001.*